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# Effect of the unpolarized spin state in spin-correlation measurement of two protons produced in the $^{12}\text{C}(d, ^2\text{He})$ reaction

S Hamieh

Kernfysisch Versneller Instituut, Zernikelaan 25, 9747 AA Groningen, The Netherlands

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## Abstract

In this paper we discuss the effect of the unpolarized state in the spin-correlation measurement of the  $^1S_0$  two-proton state produced in the  $^{12}\text{C}(d, ^2\text{He})$  reaction at the Kernfysisch Versneller Instituut (KVI), Groningen. We show that in the presence of the unpolarized state the *maximal* violation of the Clauser, Horne, Shimony and Holth (CHSH)–Bell inequality is lower than the classical limit if the purity of the state is less than  $\sim 70\%$ . In particular, for the KVI experiment the violation of the CHSH–Bell inequality should be corrected by a factor of  $\sim 10\%$  from the pure  $^1S_0$  state.

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## 1. Introduction

In an experiment performed at the Kernfysisch Versneller Instituut (KVI), Groningen [5] with the goal to test Bell inequality violation in nuclear physics (perhaps to be applied in quantum information physics), the experimental group, by bombarding a  $^{12}\text{C}$  target with 170 MeV  $d$ , was able to generate a singlet spin, two-proton state coupled to unpolarized state with  $\sim 10\%$  contribution. In this paper we analyse the experimental results of this experiment and show that the effect of the unpolarized state is important and could not be neglected.

## 2. CHSH inequalities and entanglement in a mixed ensemble

Bell-type inequalities relating averages of four random dichotomic variables  $a, a'$  and  $b, b'$  represent measurements of spin in directions  $\hat{a}, \hat{a}'$  and  $\hat{b}, \hat{b}'$ . The Clauser, Horne, Shimony and Holth (CHSH) [3] form of Bell-type inequalities for spin-1/2 case could be written in the form

$$|E(\phi_1, \phi'_1, \phi_2, \phi'_2)| = |E(\phi_1, \phi_2) + E(\phi_1, \phi'_2) + E(\phi'_1, \phi_2) - E(\phi'_1, \phi'_2)| \leq 2 \quad (1)$$

where  $\phi_i$  is the analyser angular setting for the  $i$ th particles ( $i = 1, 2$ ) and  $E(\phi_i, \phi_j)$  is the correlation function defined as

$$E(\phi_i, \phi_j) = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{\text{total}}}. \tag{2}$$

In quantum-theory language the CHSH operator corresponding to the CHSH inequality is represented by an operator

$$\mathcal{B} = \hat{a} \cdot \sigma \otimes (\hat{b} + \hat{b}') \cdot \sigma + \hat{a}' \cdot \sigma \otimes (\hat{b} - \hat{b}') \cdot \sigma \tag{3}$$

acting in Hilbert space  $\mathcal{H}_A \otimes \mathcal{H}_B$  in  $2 \otimes 2$  dimension. The correlation function is given by the mean value of the operator  $\hat{a}\sigma \otimes \hat{b}\sigma$ . For a pure state this correlation function could be easily computed, e.g. for singlet state we have

$$E(\phi_i, \phi_j) = -\cos(\phi_i - \phi_j). \tag{4}$$

For mixed state, however, the mean value should be averaged over the ensemble and therefore the CHSH inequality is not a sufficient condition to test the presence of entanglement [7]. Different measures of the entanglement have been proposed in the literature for mixed state<sup>1</sup>, e.g. entanglement of formation, distillation, relative entropy of entanglement, negativity, etc. Here we will use the entanglement of formation as our measure of the entanglement.

In a mixed ensemble any bipartite quantum state  $\rho_{AB}$  can be written as

$$\rho_{AB} = \frac{1}{4} \left( I \otimes I + \mathbf{A} \cdot \sigma \otimes I + I \otimes \mathbf{P} \cdot \sigma + \sum_{i,j=1}^3 D_{ij} \sigma_i \otimes \sigma_j \right). \tag{5}$$

$\sigma_i$  are the pauli matrices,  $I$  is the identity operator and  $\mathbf{A}$  and  $\mathbf{P}$  are vectors in  $\mathcal{R}^3$ . The  $D_{ij}$  form a  $3 \times 3$  matrix called the correlation matrix  $D$ . In this representation of the density matrix the mean value of the CHSH–Bell operator is given by [4]

$$\langle \mathcal{B} \rangle = \hat{a} \cdot [D(\hat{b} + \hat{b}')] + \hat{a}' \cdot [D(\hat{b} - \hat{b}')]. \tag{6}$$

Using the representation of the density matrix given in equation (5), we characterize any bipartite quantum state  $\rho_{AB}$  by the following.

- The entanglement measured by the ‘tangle’,  $\tau$ , of the entanglement of formation [6] and defined by

$$\tau = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\} \tag{7}$$

where the  $\lambda$  are the square roots of the eigenvalues, in decreasing order, of the matrix  $\rho_{AB}(\sigma_y \otimes \sigma_y \rho_{AB}^* \sigma_y \otimes \sigma_y)$  and  $\rho_{AB}^*$  is the complex conjugation of  $\rho_{AB}$  in the computational basis  $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$ .

- The maximum amount of the CHSH–Bell violation of the state  $\rho_{AB}$  [4]

$$\langle \mathcal{B} \rangle^{\text{max}} = 2\sqrt{M(\rho_{AB})}. \tag{8}$$

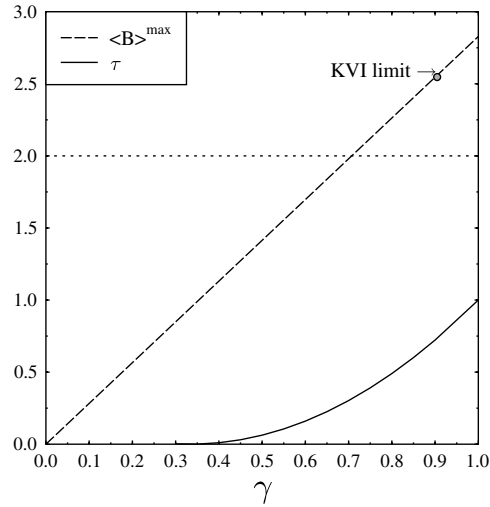
$M(\rho_{AB})$  is the sum of the two larger eigenvalues of  $DD^\dagger$ .<sup>2</sup>

- The purity of the state that measures how far the state is from pure state

$$S_L = \text{Tr}(\rho_{AB}^2). \tag{9}$$

<sup>1</sup> Any measurement of the entanglement should not increase by local operation (e.g. unitary transformation) and classical communication (e.g. phone calls), known as LOCC.

<sup>2</sup> In this case the directions  $\hat{b}$  and  $\hat{b}'$  of the analyser setting are equal to  $\cos(\theta)\hat{c}_{\text{max}} \pm \sin(\theta)\hat{c}'_{\text{max}}$  and the directions  $\hat{a}, \hat{a}'$  are equal to  $\frac{D\hat{c}_{\text{max}}}{\|D\hat{c}_{\text{max}}\|}, \frac{D\hat{c}'_{\text{max}}}{\|D\hat{c}'_{\text{max}}\|}$ , respectively.  $\hat{c}_{\text{max}}$  and  $\hat{c}'_{\text{max}}$  are two unit (not unique) and mutually orthogonal vectors in  $\mathcal{R}^3$  that maximize the function  $\|D\hat{c}\|^2 + \|D\hat{c}'\|^2$  (see [4] for more detail).



**Figure 1.** Plot of  $\langle \mathcal{B} \rangle_{\text{Werner}}^{\text{max}}$  (dashed line) and  $\tau$  (solid line) versus the purity  $\gamma$ . The dotted line is the Bell limit, the circle is the KVI limit for  $\gamma \sim 0.9$ .

### 3. Analysis of the experimental data of the KVI experiment

The spin state of the two protons produced in the  $^{12}\text{C}(d,^2\text{He})$  reaction at  $E_d = 170$  MeV at KVI [5] is a singlet state mixed with the unpolarized (random contamination) state with  $\gamma$  ( $0 \leq \gamma \leq 1$ ) controlling the degree of mixing (the details of the experimental setup and analysis method of the  $(d,^2\text{He})$  reaction were described in detail in [5]). Given all that, we can write the density matrix of such state as

$$\rho_{\text{W}} = (1 - \gamma) \frac{I}{4} + \gamma |\Psi^-\rangle \langle \Psi^-| \quad (10)$$

which interpolates between the unpolarized state  $I/4$  and singlet state  $|\Psi^-\rangle = (|+-\rangle - |-+\rangle)/\sqrt{2}$ . This class of states is called Werner states [7]. The purity of Werner states is a monotonic function of  $\gamma$ . Thus, in this paper we use  $\gamma$  as our measure of purity. Also, for Werner state it is easy to prove, using the condition noted above, that

$$\langle \mathcal{B} \rangle_{\text{Werner}}^{\text{max}} = \gamma \langle \mathcal{B} \rangle_{\text{pure}}^{\text{max}}. \quad (11)$$

Note that a violation of the modified Bell inequality does not exclude an explanation with a hidden variable theory. In figure 1 we plot  $\langle \mathcal{B} \rangle_{\text{Werner}}^{\text{max}}$  and the tangle  $\tau$  versus the purity  $\gamma$ . As we can see in this figure the Werner state does not violate the Bell inequality if its purity  $\gamma$  is less than  $1/\sqrt{2} \sim 70\%$ . However, the entanglement is still non-zero in the Werner state until  $\gamma > 1/3 \sim 33\%$ . Therefore, some quantum correlations cannot be seen only by measuring the violation of the Bell-type inequality because some of them (Werner states) are entangled but still do not violate the Bell inequality. Note that there is a possible experimental measurement of the entanglement based on the entanglement witness [8] that we think to implement in the future experiment.

In table 1 we compare the quantum theory predictions assuming a pure singlet state (case 1) and mixed Werner states (case 2) for the spin of the two detected protons for several violating cases of the CHSH–Bell inequality. The value of  $\chi^2 = \sum_i [(R_{\text{th}}^i - R_{\text{exp}}^i)/\Delta R_{\text{exp}}^i]^2$  is given at the bottom of the table for both cases. We have found that  $\chi_{\text{Werner}}^2 < \chi_{\text{singlet}}^2$  as

**Table 1.** Experimental data and quantum theory predictions for a pure singlet state (case 1) and mixed Werner states (case 2) for several violating cases of the CHSH–Bell inequality according to the definition given in equation (1).

CHSH–Bell inequality	QM		Experimental data
	Case 1	Case 2	
$E(50^\circ, 0^\circ, 25^\circ, 75^\circ)$	2.46	2.21	$0.67 \pm 2.30$
$E(60^\circ, 0^\circ, 30^\circ, 90^\circ)$	2.60	2.34	$1.21 \pm 2.42$
$E(70^\circ, 0^\circ, 35^\circ, 105^\circ)$	2.72	2.45	$1.54 \pm 2.76$
$E(80^\circ, 0^\circ, 40^\circ, 120^\circ)$	2.80	2.52	$2.11 \pm 2.86$
$E(90^\circ, 0^\circ, 45^\circ, 135^\circ)$	2.83	2.55	$2.23 \pm 2.48$
$E(100^\circ, 0^\circ, 50^\circ, 150^\circ)$	2.79	2.51	$2.39 \pm 2.87$
$E(110^\circ, 0^\circ, 55^\circ, 165^\circ)$	2.69	2.34	$2.58 \pm 2.91$
$E(120^\circ, 0^\circ, 60^\circ, 180^\circ)$	2.50	2.25	$2.75 \pm 2.95$
$\chi^2$	1.26	0.85	

expected. However, we cannot judge this result as evidence of the mixing of the singlet with the unpolarized state because the experimental data have many errors.

#### 4. Conclusion

In this paper we have discussed the effect of the unpolarized state in the spin correlation measurement of the  $^1S_0$  two-proton state produced in the  $^{12}\text{C}(d, ^2\text{He})$  reaction at KVI. We have shown that even a small coupling (less than 10%) of the pure singlet state with the unpolarized state changes dramatically the Bell-violation value. After introducing the contribution of the unpolarized state we found a better  $\chi^2$ . The experimental results have a large statistical error and therefore not conclusive for testing Bell's inequality, but with a modified experimental setup, measurements with significantly improved precision will become feasible.

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